RELATIONSHIPS BETWEEN NONLINEARITY PARAMETERS

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The nonlinearity parameters discussed by this panel can be interrelated analytically. On logarithmic scales straight line relationships result. Good agreement is found between predicted and observed quantities for power levels below the region where the gain deviates severely from its low level value.

The voltage transfer function across a nonlinear device (or system) can be approximated by:

$$e_o(t) = a_o + a_1 e_i(t) + a_2 e_i^2(t) + a_3 e_i^3(t) \ldots + j\left[b_1 e_i(t) + b_2 e_i^2(t) + b_3 e_i^3(t) + \ldots\right]$$

where: $e_i(t)$, $e_o(t)$ = time dependent input, output voltages; a_n, b_n = frequency independent amplitude and phase coefficients (small angle approximation). Assume: $n < 4$, $a_3 < 0$ (third order, compressive device) with less than octave bandwidth, then only a_1, a_3, b_1, b_3 remain to be considered.

Consider amplifiers matched to $Z_{in} = Z_{out} = (50+j0)\Omega$ so that $g_o = a_1^2$ = mag, the maximum available low level power gain. Then, for a test signal $e_i(t) = A\cos\alpha t$, the output power level at the 1 dB gain compression point is:

$$P_{\alpha,1dB}(\text{dBm}) = G_o - 10\log_{10}(|a_3|/a_1) + 0.62\text{dB}.$$

Thus, a_1 and a_3 are found from a measurement of $P_{\alpha,out}$ versus $P_{\alpha,in}$. Shifting the time reference to the output yields $b_1 = 0$ at the center frequency; from an AM-PM conversion test: $b_3 = \Delta\phi_\alpha/\Delta p_\alpha$ (radians/Watt).

Testing with two tones: $e_i(t) = A\cos\alpha t + B\cos\beta t$ and $A = B$ yields additional inband frequencies of the type $(2\alpha-\beta)$. For $A^2 << (4a_1/3|a_3|)$, a plot of $P_{\alpha,out}$ and $P_{(2\alpha-\beta),out}$ versus $P_{i(2\alpha-\beta)}$ (dBm) is related to single tone quantities by:

$$P_{i(2\alpha-\beta)} = P_{\alpha,1dB} + 10.65\text{ dB}$$

or

$$P_{i(2\alpha-\beta)} = 30\text{ dBm} - 10\log_{10}b_3$$

depending on which of the two contributions dominates.

Testing with three tones: $e_i(t) = A\cos\alpha t + B\cos\beta t + C\cos\gamma t$ and $A = B = C$ yields new output frequencies of the type $\alpha+\beta-\gamma$, 6 dB above the $2\alpha-\beta$ type. Define:

$$M_{\alpha+\beta-\gamma} = P_{\alpha+\beta-\gamma} - 3P_{\alpha,\text{ out}},$$

a constant for this model, then:

$$M_{\alpha+\beta-\gamma} = [-2P_{\alpha,1dB} - 15.25\text{dB}] \oplus [20\log_{10}b_3 - 53.9\text{dB}] = -2P_{i(2\alpha-\beta)} + 6.0\text{ dB},$$

where $\oplus$ indicates power addition.

Noise loading tests measure NPR, the ratio of output power density, $P_{n\text{ out}}$, over the desired bandwidth, Δf, to intermodulation noise power scattered into a notch of witdh $\Delta f_n << \Delta f$ at the band center. For Gaussian noise and NPR > 40 dB:

$$M_{\alpha+\beta-\gamma} = 4.3\text{ dB} - 2P_{n\text{ out}} - \text{NPR}.$$